

Problem 4

Probably the easiest of all PDEs to solve is the equation

$$\frac{\partial u(x, y)}{\partial x} = 0$$

Can you solve this equation? (Find all functions $u(x, y)$ that satisfy it.)

Solution

$$\frac{\partial u(x, y)}{\partial x} = 0$$

Integrate both sides partially with respect to x to undo the partial derivative on the left side.

$$\int^x \frac{\partial u(x', y)}{\partial x'} dx' = \int^x 0 dx'$$

Use the fundamental theorem of calculus on the left. Evaluate the integral on the right.

$$u(x, y) = 0 + f(y)$$

Instead of an arbitrary integration constant C , there is an arbitrary function f of the other variable y on the right side. Therefore,

$$u(x, y) = f(y).$$